Pseudo-Casimir effect in untwisted chiral nematic liquid crystals

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We investigate theoretically the pseudo-Casimir force between parallel plates immersed in a chiral nematic liquid crystal. We focus on small-separation limit where the director configuration between the plates inducing strong homeotropic anchoring is uniform. We find that the force is attractive at separations smaller than the crossover distance and repulsive otherwise, and that it diverges logarithmically at the critical distance where the uniform structure is replaced by a distorted structure. We also analyze modifications to the force introduced by magnetic field and comment on the possible detection of the effect.

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I. INTRODUCTION

In the past decade, the liquid-crystalline analog of the Casimir effect [1] induced by thermal fluctuations of the order parameter has been investigated theoretically in some detail [2–10]. Of all mesophases, the nematic phase attracted most attention by far, and nowadays it is well established that the phenomenology of the effect in real systems is much more intricate than the universal power law [2,3] that can, in fact, be derived by dimensional analysis. One source of deviations from the universal force profile is related to nonideality of the confining surfaces that are invariably characterized by finite rather than infinite anchoring strength [6,10] and often cannot be considered smooth [4,5]. Putting a nematic liquid crystal in an external field such as magnetic field adds another parameter to the system, and the response depends on whether the field suppresses or enhances the fluctuations. In the former case, the fluctuation-induced force becomes short range, i.e., weaker [6], whereas in the latter case, the field-driven behavior is more complex. At small separations, the attractive interaction is enhanced by the magnetic field, but at separations close to the Fréedericksz threshold, it becomes repulsive and divergent. This kind of behavior was shown to be characteristic for frustrated systems where the liquid crystal is controlled by competing external forces. Another example of a frustrated system is the so-called hybrid cell where the frustration is brought about by a mismatch of the easy axes and/or anchoring strengths [8,9].

In this paper, we extend the theory of fluctuation-induced force in frustrated nematics from geometries with competing bulk and surface fields to chiral systems where the frustration results from incompatibility of the intrinsic chiral ordering and the boundary conditions. In bulk, chiral nematic liquid crystals form a helical structure described by a wave number q_0 [11]. However, this twisted structure is suppressed in confined geometries with strong enough unidirectional anchoring. In such systems, there exists a thickness below which the stable solution is the uniform untwisted state [12–15]. The transition between the uniform and the twisted structure may be induced by varying the parameters of the system such as external fields, film thickness, and chirality. Depending on the boundary conditions and the elastic constants, the transition may be either continuous or discontinuous [14,15]. At distances larger than the critical thickness, where the low-energy configuration corresponds to a spiraling director field, pattern formation has been observed. In case of strong homeotropic anchoring, the one-dimensional chiral nematic, such as translationally invariant configuration (TIC) and periodic, double twist configuration patterns have been studied in detail [12,14]. Within the one-constant approximation, a TIC distortion occurs at a critical distance equal to half the pitch π/q_0. However, it has been shown that the energy of a periodic solution where the axis of the helix is tilted away from the plate normal is slightly lower than the energy of the TIC solution. The continuity of the structural transition depends on the relative magnitude of the Frank constants and the transition is continuous in the one-constant approximation.

Here, we would like to determine how the pseudo-Casimir effect behaves on approaching the structural transition in a confined chiral nematic liquid crystal. Although the normal modes bear the signature of the intrinsic helical distortion present in bulk, we will be able to reformulate the problem and take advantage of the mathematical procedure used in the vicinity of the Fréedericksz transition in ordinary nematic liquid crystals [8]. We restrict the analysis to the one-constant Frank elasticity theory, assume a uniform degree of order, and neglect the chirality-induced biaxial ordering.

In Sec. II, we introduce the formalism and calculate the fluctuation-induced force. Then we consider the case when the magnetic field is applied (Sec. III). In Sec. IV, we summarize our results and discuss the possibilities for experimental detection of our predictions.

II. FLUCTUATION-INDUCED FORCE

We consider a system of two parallel flat plates immersed in a chiral nematic liquid crystal with boundary conditions

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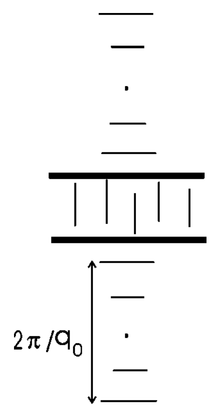

FIG. 1. Configuration of the system. In the confined section the director structure is homeotropic due to strong anchoring, whereas in the semi-infinite part of the setup it twists.

characterized by strong homeotropic anchoring (Fig. 1). The director field between the plates is homeotropic for separations smaller than the critical separation $h_c=\pi/q_0$, where q_0 is the intrinsic wave number of the helical structure [12–15].

In the homeotropic configuration, the director field can be written as $\mathbf{n}(\mathbf{r})=\mathbf{n}_0+\delta\mathbf{n}(\mathbf{r})$, where $\mathbf{n}_0=\hat{\mathbf{z}}$ is the average director field, $\delta\mathbf{n}(\mathbf{r})=n_x(\mathbf{r})\hat{\mathbf{x}}+n_y(\mathbf{r})\hat{\mathbf{y}}$ is the fluctuating director field and $\mathbf{r}$ denoting the observation point in an xyz Cartesian coordinate system. Associated with equilibrium director field in the strong anchoring case is the Frank elastic free energy [11]

$$F=\frac{K}{2}\int[(\boldsymbol{\nabla}\cdot\mathbf{n})^2+(\mathbf{n}\cdot\boldsymbol{\nabla}\times\mathbf{n}+q_0)^2+(\mathbf{n}\times\boldsymbol{\nabla}\times\mathbf{n})^2]dV, \tag{1}$$

where K is the effective elastic constant. By expanding the Frank free energy around the ground state, we derive the harmonic Hamiltonian of the two modes $n_x=n_x(\mathbf{r})$ and $n_y=n_y(\mathbf{r})$

$$H[n_x,n_y]=\frac{K}{2}\int[(\nabla n_x)^2+(\nabla n_y)^2+2q_0(n_y\partial_z n_x-n_x\partial_z n_y)]dV. \tag{2}$$

The translational invariance in the xy plane allows for using the Fourier transform of n_x and n_y: $n_w(\mathbf{r})=\Sigma_{\mathbf{p}}n_w(z;\mathbf{p})\exp(i\mathbf{p}\cdot\boldsymbol{\rho})$ where w is either x or y and $\boldsymbol{\rho}=x\hat{\mathbf{x}}+y\hat{\mathbf{y}}$. After integration over x and y the Hamiltonian reads

$$H[n_x,n_y]=\frac{KA}{2}\sum_{\mathbf{p}}\int_0^h[|\partial_z n_x|^2+|\partial_z n_y|^2+p^2(|n_x|^2+|n_y|^2)+2q_0(n_y\partial_z n_x^*-n_x\partial_z n_y^*)]dz, \tag{3}$$

where $p^2=p_x^2+p_y^2$, A is the area of each plate, h is the plate separation, and the asterisk denotes the complex conjugate.

Assuming strong anchoring [16] on the plates, the partition function of the system in analogy with quantum-mechanical path integral is

$$Z=\int\mathcal{D}n_x\int\mathcal{D}n_y\exp(-H[n_x,n_y]/k_BT), \tag{4}$$

where n_x and n_y are subject to constraints $n_x(z=0)=n_x(z=h)=n_y(z=0)=n_y(z=h)=0$, k_B is the Boltzmann constant, and T is the temperature. We stress that this partition function is formally identical to the kernel of the propagator of a two-dimensional isotropic quantum-mechanical oscillator in a magnetic field [17].

However, the transformation [18,19]

$$\phi_1=\cos(q_0z)n_x+\sin(q_0z)n_y, \tag{5}$$

$$\phi_2=-\sin(q_0z)n_x+\cos(q_0z)n_y, \tag{6}$$

diagonalizes the Hamiltonian:[1]

$$H=\frac{KA}{2}\sum_{\mathbf{p}}\sum_{w=1,2}\int_0^h[(\partial_z\phi_w)^2+(p^2-q_0^2)\phi_w^2]dz. \tag{7}$$

Now we see that this system is equivalent to the pre-Fréedericksz transition geometry studied in Ref. [8]. We find [20]

$$Z\propto\prod_{p<q_0}[\sin(\sqrt{q_0^2-p^2}h)]^{-1}\prod_{p>q_0}[\sinh(\sqrt{p^2-q_0^2}h)]^{-1}. \tag{8}$$

Having calculated the partition function, we have to identify the interaction part of the free energy. To this end, we note that the Casimir interaction free energy is measured relative to the free energy of the corresponding reference bulk state. Let us, for the moment, neglect the long-wavelength q_0^{-1} differences between the bulk nematic and the bulk chiral nematic states, and analyze the different terms of $F=-k_BT\ln Z$ on dimensional grounds. As expected, we find a term proportional to Ah, which represents the bulk free energy (actually, as we will see later on, a part of it), then there is a term proportional to A and independent of the separation, which obviously corresponds to the surface free energy, and the rest is the fluctuation-induced interaction

$$F_{int}=\frac{Ak_BT}{2\pi}\left[\int_0^{q_0}\ln\sin(p_<h)\,p_<dp_<+\int_0^\infty\ln(1-\exp(-2p_>h))p_>dp_>\right], \tag{9}$$

where we have replaced $\Sigma_{\mathbf{p}}$ by $[A/(2\pi)^2]\int d^2\mathbf{p}$. The first term in the above expression is a contribution arising from the intrinsic chirality. The second term is the standard Casimir free energy of an undistorted nematic, $-Ak_BT\zeta(3)/8\pi h^2$ where $\zeta(3)=1.202,\ldots,$ is the Riemann zeta function. A similar interaction has been discussed in two other frustrated systems, a hybrid-aligned nematic cell and a pre-Fréedericksz transition system [8].

[1]Meanwhile, in relation (7) the fluctuating fields are transformed to real bases.

The structural force, $\mathcal{F}=-\partial F/\partial h$, turns out to be attractive at small distances

$$\mathcal{F}_{fluct}(q_0 h \ll 1) \approx -\frac{k_B T A}{4\pi}\left[\frac{\zeta(3)}{h^3}+\frac{q_0^2}{h}\right], \tag{10}$$

whereas at distances comparable to the critical separation the force becomes repulsive

$$\mathcal{F}_{fluct}(q_0 h \to \pi) \approx -\frac{k_B T A q_0^3}{2\pi^2}\ln(2\sin(q_0 h)) \tag{11}$$

and diverges logarithmically at the threshold, $h_c=\pi/q_0$. The divergence of the Casimir force may be directly related to the critical fluctuations characteristic for the continuous structural transition [8].

In case of homogeneous boundary conditions, the geometry of the system is of course different, the main difference being that the uniform-twisted transition is discontinuous (due to the quantization of the pitch caused by the homogeneous anchoring) and it is easy to show that it occurs at a smaller thickness, $h_c=\pi/2q_0$. However, the interaction free energy in the uniform texture turns out to be the same as in the homeotropic cell, implying that the fluctuation-induced force does not exhibit any particular behavior at the transition. This is, of course, expected because the transition is discontinuous.

Let us now return to fluctuations in reference bulk configuration, where the director field twists to minimize the elastic free energy. There is a clear difference between the Hamiltonians of the fluctuations in the uniform state [Eq. (7)] and in the twisted bulk state. If we write the director field as $\mathbf{n}=\mathbf{n}_0+\delta\mathbf{n}$ where

$$\mathbf{n}_0=(\cos q_0 z, \sin q_0 z, 0), \tag{12}$$

and

$$\delta\mathbf{n}=(-n_1 \sin q_0 z, n_1 \cos q_0 z, n_2) \tag{13}$$

with $n_1=n_1(\mathbf{r})$ and $n_2=n_2(\mathbf{r})$ being the amplitudes of the fluctuations in directions perpendicular to n_0, the Hamiltonian of the reference bulk configuration reads

$$H[n_1,n_2]=\frac{K}{2}\int [(\nabla n_1)^2+(\nabla n_2)^2+q_0^2 n_2^2 - 4q_0 n_1(\partial_x n_2 \cos q_0 z+\partial_y n_2 \sin q_0 z)]dV. \tag{14}$$

Note that now H includes two coupled oscillators, one of which is massive as witnessed by the presence of the $q_0^2 n_2^2$ term. Qualitatively, the situation is similar as in the pre-Fréedericksz geometry [8], and we can estimate the free energy associated with the difference between the Hamiltonians by

$$\frac{k_B T A q_0^3 h}{6\pi}, \tag{15}$$

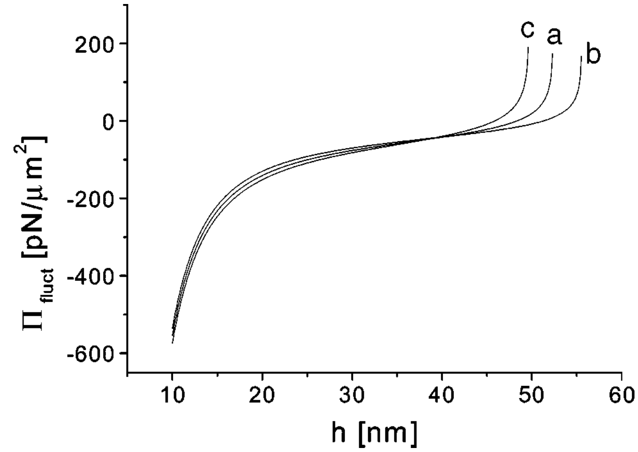

FIG. 2. Pseudo-Casimir pressure, $\Pi_{fluct}=\mathcal{F}_{fluct}/A$, in chiral nematic liquid crystal as a function of h. The intrinsic chirality q_0 is set to 6×10^7/m and $T=300$ K. The attractive tail becomes repulsive as the distance is increased, the crossover being located at (a) $h=48$ nm. At the structural transition, the force diverges logarithmically. In the presence of a magnetic field (Sec. III) in the case of (b) positive and (c) negative diamagnetic susceptibility, the crossovers occur at 51 nm and 46 nm respectively where $\xi^{-1}=2\times10^7$/m.

as if there were two massive modes with correlation length q_0^{-1}. This gives rise to an additional attractive force of $-k_B T A q_0^3/6\pi$ and thus enhances the fluctuation-induced interaction in the system. This extra attraction is dominant at intermediate distances: at h's smaller than $\sim 2.9/q_0$, the effects of chirality are not very prominent and $\mathcal{F}$ is essentially proportional to h^{-3}, whereas at h's larger than $\sim 2.9/q_0$, the force profile is determined by the pretransitional slowdown of the normal modes (Fig. 2, curve a).

Finally, we note that the uniformity of the director configuration between the plates gives rise to an elastic free energy

$$\frac{1}{2}KAq_0^2 h, \tag{16}$$

which results in a mean-field, thickness-independent attraction between the plates, analogous to the mean-field attraction caused by the magnetic field in the Fréedericksz cell [8]. This force is considerably stronger in magnitude than the fluctuation-induced force: the ratio of their magnitudes, at distances comparable to q_0^{-1}, may be estimated by $k_B T q_0/K$ so that for a chiral nematic system with a pitch of the order of 100 nm at room temperature, the fluctuation-induced force amounts to less than a percent of the mean-field force. However, this does not mean that the pseudo-Casimir effect cannot be detected: the strong mean-field background force is independent of thickness, whereas the fluctuation-induced force has a thickness-dependent component. Thus, a technique sensitive to variations superposed to a constant background could be used to study the fluctuation-induced force.

III. CHIRAL NEMATIC LIQUID CRYSTAL IN EXTERNAL FIELD

It is well known that the structural transition in chiral nematics may be easily induced by an external field [21,22]: most electrooptic applications of liquid crystals are based on this phenomenon. With the progress of miniaturization of these devices, there may soon be a time when interactions described in this study will become important for practical purposes. With this motivation, let us consider the system of a chiral nematic liquid crystal in the presence of an external field. To avoid possible effect of electrical conduction and other technical details, we limit our discussion to a constant magnetic field in the case of (*A*) positive and (*B*) negative anisotropy of diamagnetic susceptibility. A magnetic field **B** normal to the plates will destabilize the homeotropic structure with respect to the twisted structure if the anisotropy of the susceptibility is negative; in materials with positive anisotropy, the field along the director will have a stabilizing effect. The magnetic contribution to the free energy [11] reads

$$-\frac{\chi_a}{2\mu_0}\int(\mathbf{B}\cdot\mathbf{n})^2 dV, \tag{17}$$

where χ_a is the anisotropy of the diamagnetic susceptibility and μ_0 is the magnetic permeability of vacuum. Therefore, the inclusion of the magnetic field leads to the Hamiltonian

$$H[n_x,n_y]=\frac{K}{2}\int[(\nabla n_x)^2+(\nabla n_y)^2\pm\xi^{-2}(n_x^2+n_y^2)+2q_0(n_y\partial_z n_x-n_x\partial_z n_y)]dV, \tag{18}$$

where $\xi=\sqrt{K\mu_0/|\chi_a|B^2}$ is the magnetic coherence length, and "+" and "−" refer to positive and negative χ_a, respectively.

A. Positive χ_a

The diagonalized Hamiltonian in Fourier space reads

$$H=\frac{KA}{2}\sum_{\mathbf{p}}\sum_{w=1,2}\int_0^h\{(\partial_z\phi_w)^2+[p^2-(q_0^2-\xi^{-2})]\phi_w^2\}\,dz. \tag{19}$$

In the weak magnetic field limit $\xi^{-1}<q_0$, the qualitative behavior of the system and the pseudo-Casimir force is the same as if there were no magnetic field. The presence of the field induces an effective chirality $q_{eff}=\sqrt{q_0^2-\xi^{-2}}$ smaller than the intrinsic chirality, which stabilizes the homeotropic alignment preferred by the anchoring and makes the critical separation increase to a value of $h_c=\pi/\sqrt{q_0^2-\xi^{-2}}$. The magnetic field makes the force weaker (Fig. 2, curve b).

It is interesting to note that when ξ^{-1} approaches q_0, h_c increases and right at $\xi^{-1}=q_0$ it becomes infinite. This is, in fact, the critical value of ξ^{-1} where a field-driven structural transition from field-induced homeotropic state to planar state (the pitch normal to the plates), through a TIC configuration, takes place [21,22].

In the strong magnetic field limit, $\xi^{-1}>q_0$, the behavior of the system is really determined by the magnetic field rather than by chirality and the uniform structure is stable at all separations. In this case, the system is no longer frustrated and the stabilizing role of the magnetic field actually suppresses the fluctuations and the normal modes become massive. The partition function of the system turns out to be given by $Z\propto\sinh(\sqrt{p^2+\xi_{eff}^{-2}}h)$, where $\xi_{eff}^{-1}=\sqrt{\xi^{-2}-q_0^2}$ is the inverse effective magnetic coherence length. The interaction free energy reads

$$F_{int}=\frac{k_BTA}{2\pi}\int_{\xi_{eff}^{-1}}^{\infty}\ln(1-\exp(-2ph))p\,dp. \tag{20}$$

By expanding the logarithm in Taylor series, $\ln(1-\exp(-2ph))=-\sum_{m=1}^{\infty}(1/m)\exp(-2mph)$, and performing integration we obtain

$$F_{int}=-\frac{k_BTA}{8\pi h^2}\sum_{m=1}^{\infty}\frac{1}{m^3}\left(2m\frac{h}{\xi_{eff}}+1\right)\exp\left(-2m\frac{h}{\xi_{eff}}\right). \tag{21}$$

For massive fluctuations, the interaction free energy depends strongly on the reduced length h/ξ_{eff}. In the limit of small separations, the pseudo-Casimir force reduces to the standard Casimir force in nematics, $-k_BTA\zeta(3)/4\pi h^3$, whereas the asymptotic behavior of the force for $h\gg\xi_{eff}$ is given by $-k_BTA\exp(-2h/\xi_{eff})/2\pi\xi_{eff}^2h$. This is a short-range force, analogous to the interaction induced by fluctuations of the degree of order and biaxial modes in nematic mesophase [6].

B. Negative χ_a

When $\chi_a<0$, a magnetic field perpendicular to the cell always has a destabilizing effect, i.e., the field tends to rotate the director towards a planar orientation [11] and thus facilitates the uniform-twisted transition. In this case, the Hamiltonian reads

$$H=\frac{KA}{2}\sum_{\mathbf{p}}\sum_{w=1,2}\int_0^h\{(\partial_z\phi_w)^2+[p^2-(q_0^2+\xi^{-2})]\phi_w^2\}dz. \tag{22}$$

The system is obviously equivalent to the zero-field geometry with a larger effective chirality $q_{eff}=\sqrt{q_0^2+\xi^{-2}}$ and accordingly shorter critical distance $h_c=\pi/\sqrt{q_0^2+\xi^{-2}}$. In this case, the Casimir force is enhanced by the magnetic field (Fig. 2, curve c).

C. Mean-field force

Once the chiral nematic is subjected to a magnetic field, there are two bulk-aligning terms: the chirality and the magnetic field. The combined effect of chirality and magnetic field give rise to an effective mean-field interaction.

In the case of positive anisotropy of diamagnetic susceptibility, the chiral nematic in bulk twists in such a way that the helical axis is normal to the field, i.e., parallel to the substrates (Fig. 3). With increasing magnetic field, the helix

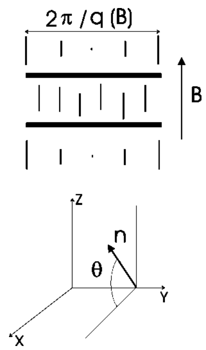

FIG. 3. Director configuration of the system in the case $\chi_a>0$: In bulk, the director twists with a helical axis perpendicular to **B**, thereby reducing the magnetic field energy.

becomes more and more distorted and its pitch increases. At the critical field, i.e., $\xi_c^{-1}=\pi q_0/2$, the helix unwinds completely and the pitch diverges [11].

The free-energy density in the bulk is given by

$$f_{MF}=\frac{K_2}{2}\left[\left(q_0-\frac{\partial\theta}{\partial y}\right)^2-\xi^{-2}\sin^2\theta\right], \quad (23)$$

where $\theta=\theta(y)$ denotes the orientation of the director field as shown in Fig. 3. The first integral of the Euler–Lagrange equation gives

$$\frac{d\theta}{dy}=-(C-\xi^{-2}\sin^2\theta)^{1/2} \quad (24)$$

where the integration constant C is determined by minimizing the free energy: C satisfies the condition

$$\sqrt{C}E(\xi^{-2}/C)=\pi q_0/2, \quad (25)$$

where $E(\xi^{-2}/C)$ is the elliptic integral of the second kind.[2] Here we examine the value of the C in two limiting cases: for $\xi^{-1}=0$, $d\theta/dy=-q_0$, therefore from Eq. (24) we obtain $C=q_0^2$ while for $\xi^{-1}\geqslant\xi_c^{-1}$, $d\theta/dy=0$, $\theta=\pi/2$, and so $C=\xi^{-2}$.

It may be shown that the bulk free-energy per unit volume reads [23]

$$\frac{F_{MF}}{V}=\frac{K}{2}(q_0^2-C) \quad (26)$$

and so the mean-field force per unit area is

$$\frac{\mathcal{F}_{MF}}{A}=-\frac{K}{2}(C-\xi^{-2}). \quad (27)$$

We note that the value of C increases from q_0^2 to ξ_c^{-2} when the applied field ξ^{-1} increases from zero to ξ_c^{-1}, so that the mean-field force is attractive in the range $0\leqslant\xi^{-1}<\pi q_0/2$ and its magnitude decreases with increasing field. Beyond the unwinding critical value ξ_c^{-1}, where the uniform director structure is established, the mean-field force vanishes. For ξ^{-1} between 2×10^7/m and 5×10^7/m, the corresponding decrease of the mean-field force is between 10 to 30% where q_0 is set to 6×10^7/m. A weak mean-field force may be advantageous in experiments designed to detect the fluctuation-induced force.

In the case where $\chi_a<0$, the helical structure outside the plates is planar with the director perpendicular to the magnetic field. Thus, the effective mean-field force is given by $-KAq_{eff}^2/2$, where $q_{eff}^2=q_0^2+\xi^{-2}$.

IV. CONCLUDING REMARKS

In this study, we have analyzed the fluctuation-induced structural force in untwisted chiral nematic liquid crystals. We have demonstrated that the system shares the main features of this interaction—an enhanced attraction at small separation compared to nonfrustrated systems, a crossover from attraction to repulsion at separations comparable to the critical distance, and pretransitional divergence—with other frustrated liquid-crystalline geometries [8]. Thus, we have established a unified picture of the pseudo-Casimir force in nematic geometries subjected to competing surface, bulk, and/or internal fields.

Although the three frustrated geometries discussed so far (hybrid cell, Fréedericksz cell, and chiral nematic cell) basically all contain the same physics, there are important differences between them. For example, the only control parameter for the hybrid cell is the temperature, and it may well turn out that the experimentally accessible range of variation of the anchoring strength with temperature is rather narrow. The chiral nematic cell, on the other hand, has another control parameter, the intrinsic pitch, which may vary considerably upon changing the temperature, thereby providing more flexibility in experiments. When the system is placed in a magnetic field, the field may—depending on its strength and orientation—either enhance or suppress the frustration resulting from chirality and the boundary conditions. As a result, it should be easier to design an experiment sensitive to the fluctuation-induced interaction. In addition, the force could be studied while varying the chirality and the magnetic field at a "constant" separation.

In most experimental situations, the anisotropy in the elastic constants renders the structural transition discontinuous [14] and shifts the corresponding singularity in the Casimir force to metastable regime. Nevertheless, the fingerprint texture may be avoided by applying an external field: in some ranges of the involved parameters—namely, the film thickness and the magnetic field—a transient TIC may experimentally form [24,25]. In such a setup, even for unequal elastic constants the pretransitional divergence of the Casimir force should be detectable. In addition, polymeric liquid crystals that have large splay constants K_1 may exhibit a second-order structural transition [14]. If chiral nematic liquid-crystalline phases exist in such polymeric materials, they would be good candidates for observing phenomena discussed here.

With the development of the submicroconfined liquid-crystalline systems where the length scale of the confinement

[2] $E(e)=\int_0^{\pi/2}(1-e\sin^2\theta)^{1/2}d\theta$.

is decreasing further and further, the fluctuation forces are expected to become important. At these scales, the Casimir forces are not only measurable, they may actually be essential for several interesting phenomena such as stability of thin films [9] which may be important for a technologically very promising method of self assembly of patterned surfaces [26].

Although the existing understanding of the phenomenology of the pseudo-Casimir effect in nematic liquid crystals is fairly elaborate, there are a few outstanding problems in the field that still remain unsolved. In particular, little is known about the behavior of the force beyond the structural transition where the system is characterized by a distorted director field. While the fluctuation-induced effects in a distorted director configuration are less prominent due to the presence of the elastic mean-field interaction, they are needed for a complete description of the continuous structural transitions. In a future study, we will address this question.

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